Chapter 3

On the Design of Regional Insurance Schemes/Mechanisms for East Asia

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CHAPTER 3

On the Design of Regional Insurance Schemes/Mechanisms for East Asia

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This paper identifies the issues that would be central in designing a possible regional insurance scheme or mechanism for East Asia. The main focus is on the risk sharing mechanism for catastrophe risks households in the region incur. We apply the theoretical observations by Nakata et al. (2010) that provide a consistent explanation for the apparent anomalies concerning the demand for catastrophe insurance within the subjective expected utility framework. The key observation is that the number of observations would be inevitably insufficient to warrant a robust probability estimate for a rare event. The inherent lack of a robust probability estimate leads to diverse probability beliefs. We evaluate the various insurance schemes in terms of social welfare. In doing so, we adopt a measure that is based on the ex post *social welfare concept in the sense of Hammond (1981), since the standard Pareto optimality criterion is problematic in the presence of diverse beliefs, for it ignores the regrets or pleasure* ex post *caused by 'incorrect' beliefs. Although the* ex post *social welfare may have an expected utility form, we only focus on the* ex post *utility frontier rather than specifying a particular social probability. We postulate that a desirable insurance scheme is the one that eliminates any personal catastrophe state.*

Keywords: catastrophe, demand for insurance, diverse beliefs, ex post social welfare.

1. Introduction

East Asia has historically been hit numerous times by catastrophic natural disasters. Last year alone, the region suffered from the great earthquake and tsunami in Japan and the great flooding in Thailand. Whilst it would be impossible to prevent the occurrence of a natural disaster itself, every effort should be made to prevent and limit the level of damages natural disasters could inflict. Nevertheless, some damage or loss from natural disasters is inevitable—especially for catastrophes, which implies the need for an insurance mechanism.

Catastrophe insurance or insurance for natural disasters, however, is not very common in practice. The fact that a catastrophe typically incurs a macro risk invalidates the application of the strong law of large numbers, on which a typical insurance mechanism is based. This is the reason why catastrophe insurance is often backed or indirectly supplied by the government. Nevertheless, there is evidence that the demand for catastrophe insurance is often weak, even though the insurance premium is apparently set favourably to the (potential) buyers.

This paper examines the issues that are key to design a regional insurance scheme for catastrophes or natural disasters. The presumed target of the possible insurance schemes examined in the paper is the household sector, rather than the corporate sector. Nevertheless, many issues raised in this paper would remain valid for insurance schemes that target the corporate sector. In the analysis, we apply the theoretical explanation for the weak demand for catastrophe insurance given by Nakata, *et al*. (2010). The key observation of Nakata, *et al*. (2010) is that rare events by definition take place very infrequently, which implies that no robust probability estimate of a rare event would be readily warranted by empirical or scientific evidence. Thus, diverse probability beliefs would be inevitable, which in turn results in a weak demand for catastrophe insurance. Based on this observation, we compare several insurance schemes that differ in terms of the payment structure and also regarding the subscription structure. To be more specific, we compare conventional indemnity insurance and index insurance with respect to the payment structure, whilst we also compare direct subscription by households, subscription by local governments, and subscription by national governments with respect to the subscription structure.

Once we allow for diverse beliefs, the welfare evaluation of a regional insurance scheme is not straightforward. Since the standard Pareto optimality is based on the *ex ante* preferences of the agents that govern their decisions, the standard Pareto optimality is inherently an *ex ante* criterion. With diverse beliefs, agents would be making 'incorrect' decisions or 'mistakes', although such 'mistakes' are not understood as mistakes a priori by the agents themselves. If we know the true probability, then we may still be able to evaluate how agents are making mistakes, and consequently we may evaluate the insurance schemes with respect to the true probability. However, it is rather unreasonable to assume such knowledge, especially for rare events.

Thus, the use of the standard Pareto criterion calls for a significant value judgement, since *ex ante* preferences do not capture regrets or pleasure arising from the outcomes of decisions made in accord with incorrect subjective beliefs. Such arguments can be found in Diamond (1967) and Drèze (1970), while Starr (1973) introduces the notion of *ex post* optimality, which is based on realised allocations rather than prospects of future allocations, with which the standard *ex ante* optimality is defined. Starr (1973) shows that the two concepts do not coincide generically, when beliefs are heterogeneous. Hammond (1981) introduces the notion of the *ex post* social welfare optimum, which is based on an expected social welfare function, where the expectation is with respect to the social planner's probability (or the social probability) rather than with respect to the subjective beliefs of the agents. Hammond (1981) shows that the *ex post* social welfare optimum does not coincide with the usual *ex ante* social welfare optimum when the subjective beliefs are heterogeneous. However, the choice of the social probability for the *ex post* social welfare function is not trivial. Thus, it is important to identify the conditions under which a reasonably high level of *ex post* social welfare can be assured, even when we do not know the social probabilities.

In this regard, the absence of personal catastrophe states for everyone would assure a reasonably high level of *ex post* social welfare, as Nakata (2012) shows in a dynamic general equilibrium model with diverse beliefs.[1] Note that the existence of a personal catastrophe state is not very damaging from the *ex ante* point of view if the agent

[1] We use the term a 'personal catastrophe state' to describe a state in which an agent is left with an extremely low level of wealth.

believes that the probability of such a state is extremely low, which implies that an agent may choose to allow for the emergence of personal catastrophe states.

Based on the above observations, this paper therefore compares the various insurance schemes from the viewpoint of the *ex post* social welfare. In particular, we examine conventional indemnity insurance and index insurance. In doing so, we pay attention to the subscription structure, i.e. direct subscriptions by households, subscriptions by local governments, and subscriptions by national governments.

2. Insurance Demand under Diverse Beliefs

In this section, we first review the three stylised facts regarding demand for catastrophe insurance based on aggregate data. Then, we show that the willingness-to-pay for insurance is almost linear in the subjective loss probability when the loss probability is very small, by numerical examples. By following Nakata, *et al*. (2010), we then show that subjective loss probabilities may well be very diverse and unstable for rare loss events, which in conjunction with the almost linear property of the willingness-to-pay, provides a consistent explanation for the three stylised facts.

2.1. Stylised Facts about Catastrophe Insurance

It is well reported that there are some anomalies regarding the demand for insurance that appear to be incompatible with the standard expected utility framework. First, there is evidence that insurance for catastrophes, such as earthquakes or flooding, is not very widely purchased, even though many policies against catastrophes are subsidised by governments to keep the premiums favourable to the buyers, e.g. earthquake insurance in Japan and the National Flood Insurance Program (NFIP) in the United States.[2] In contrast, it is widely known that commonly sold insurance policies such as travel insurance, home insurance and medical insurance, have a substantial mark-up. Yet, many people voluntarily elect to purchase such policies. Hence, insurance for

[2] Kunreuther, *et al*. (1978) is one of the pioneering works that reported this "anomaly." Also, see Michel-Kerjan (2010) for the history of NFIP.

catastrophes is purchased with less frequency compared to insurance for moderate risk (e.g. travel insurance), even if the premium is often set more favourably for catastrophe insurance (stylised fact 1). Furthermore, (a) market penetration is much lower in areas that have historically been less frequently hit by catastrophes, even if the premiums are adjusted to reflect the lower frequencies (stylised fact 2), and (b) market penetration jumps up immediately after a catastrophe (stylised fact 3).[3]

The above three stylised facts present difficulties in designing a catastrophe insurance scheme that would enjoy a wide subscription. Although it appears that these facts are in contradiction with the model of insurance demand based on the standard expected utility framework, the apparent incompatibility becomes less straightforward once we allow for diverse subjective beliefs. In what follows, we introduce a simplified version of the framework by Nakata, *et al*. (2010), which attempts to explain the three stylised facts above simultaneously within the subjective expected utility framework.

2.2. The Willingness-to-pay for Catastrophe Insurance

Consider an agent who is facing some uncertainty. We assume for simplicity that there are only two states, state 1 (the no-loss state) and state 2 (the loss state), with $x > 0$ being the loss in state 2. Let W denote the initial wealth of the agent. Then, the final wealth is W in state 1 and $W - x$ in state 2 when there is no insurance.

Assume that the agent is a risk-averse expected utility maximiser, who makes some probability estimate of the two states; π is the agent's subjective probability of the no loss state. Now, assume that any loss up to b $(\leq x)$ can be covered by insurance with a premium of ρ_b. With this insurance, the final wealth becomes $W - \rho_b$ in state 1 and $W - x + b - \rho_b$. Agent h purchases the insurance if

$$\pi \cdot u(W) + (1-\pi) \cdot u(W - x) \leq \pi \cdot u(W - \rho_b) + (1-\pi) \cdot u(W - x + b - \rho_b)$$

while the agent is indifferent between purchasing and not purchasing when an equality

3 See for instance, Dixon, *et al*. (2006), and Browne & Hoyt (2000).

holds. This observation leads us to define the agent's willingness-to-pay for this insurance as $\hat{\rho}_b$, satisfying the following equation:

$$\pi \cdot u(W) + (1-\pi) \cdot u(W-x) = \pi \cdot u(W-\hat{\rho}_b) + (1-\pi) \cdot u(W-x+b-\hat{\rho}_b)$$

The willingness-to-pay is almost linear in the loss probability $(1-\pi)$ when $(1-\pi)$ is very small as the following numerical example illustrates.

Example: Effects of loss probability and degree of risk aversion

Suppose an agent possesses a property of 100, but it is subject to a damage of 30 with subjective probability π. We assume that the loss can be fully insured (i.e. $x = b = 30$), and the willingness-to-pay for such an insurance policy $\hat{\rho}^{\pi}$ is computed by

$$\hat{\rho}^{\pi} \coloneqq 100 - u^{-1}[\pi \cdot u(100) + (1-\pi) \cdot u(70)].$$

Table 1 reports the values of $\hat{\rho}^{\pi}$ for different values of loss probability π and those of the degrees of relative risk aversion γ for a power utility $u(w) = w^{1-\gamma}/(1-\gamma)$.

Table 1: Effects of Loss Probability and Degree of Risk Aversion

$1-\pi$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
γ	3.50	3.56×10^{-1}	3.57×10^{-2}	3.57×10^{-3}	3.57×10^{-4}	3.57×10^{-5}
γ	4.11	4.27×10^{-1}	4.28×10^{-2}	4.29×10^{-3}	4.29×10^{-4}	4.29×10^{-5}
γ	4.83	5.16×10^{-1}	5.20×10^{-2}	5.20×10^{-3}	5.20×10^{-4}	5.20×10^{-5}

Note that a catastrophe is typically rare, but causes a huge damage. In the example, this is characterised by a very small true loss probability $(1-\pi^*)$. The trouble is that no one really knows the true probability π^* even if we may all agree that $(1-\pi^*)$ is 'very small'. However, both 1 in 1 million (or 10^{-6}) and 1 in 10,000 (or 10^{-4}) are small probabilities, yet the willingness-to-pay for the latter is approximately 100 times of the former, regardless of the degree of relative risk aversion in the above example. Nakata, *et al*. (2010) focuses on this feature to explain the above three stylised facts within the subjective expected utility framework. In what follows, we briefly explain the argument of Nakata, *et al*. (2010).

2.3. Diverse Beliefs and Rationality

The main idea of Nakata, *et al*. (2010) is simple. It is based on the fact that rare events are by definition observed very infrequently.[4] More specifically, rare events tend to be unprecedented even if the probability is not zero; thus, the empirical relative frequency tends to be zero. This implies that both a loss probability of 1 in 1 million and that of 1 in 1 billion would be consistent with data of no loss out of 100 observations. However, an insurance premium based on a probability of 1 in 1 million and that based on a probability of 1 in 1 billion will be very different in scale. In contrast, just one occurrence of a rare event will typically result in an inflated empirical relative frequency. This is compatible with stylised fact 3, i.e. an immediate jump in the market penetration upon occurrence of a catastrophe. The key observation is that the empirical data for rare events would not be sufficiently large to warrant a robust probability estimate, which would restrict diversity in terms of insurance premium or willingness-to-pay for insurance.

We illustrate the point more specifically by the following simplified version of the model in Nakata, *et al*. (2010). Suppose there are two states for each period, where state 1 is the no-loss state and state 2 is the loss state. The sequence is known to be i.i.d. Let π denote the probability of state 1 in each period. Table 2 reports $P_\pi\{\text{no loss}\}$, the probability of observing no losses for different levels of π, when we fix the length of the sequence as 100.Observer that when π is as low as 0.9999 (i.e. the probability of the loss states is 1/10000), the probability of observing no loss for 100 observations is approximately 0.99. This means that for any $\pi \geq$ 0.9999, up to about 99% there will be no loss for 100 periods. Hence, the scale of loss probabilities that are compatible with the empirical frequency may vary substantially for rare events, particularly unprecedented events (any probability less than 1/10000 is very plausible when there

4 In the macroeconomics literature, Rietz (1988) introduced a rare disaster state to resolve the equity premium puzzle. His framework has attracted renewed attention recently—e.g., Weitzman (2007) and Barro (2009).

are 100 periods). In other words, the diversity of beliefs may well be very large in terms of the scale of the loss probability even if all agents are rational in the sense that their beliefs are compatible with the empirical data.

Table 2: Probability of Observing No Loss in 100 Period

π	$P_\pi\{\text{no loss}\}$
0.999999999	0.999999900
0.999999	0.999900050
0.9999	0.990049339
0.999	0.904792147
0.99	0.366032341

The large diversity in the subjective loss probability has a significant implication for the insurance demand. Recall that the willingness-to-pay for the insurance is almost linear in the loss probability when the loss probability is very small. Thus, a large diversity in the scale of subjective loss probability will imply a large diversity in the willingness-to-pay for the insurance. It follows that there may well be many agents whose willingness-to-pay is lower than the insurance premium offered by the insurance provider, even if the premium is set at an actuarially fair level with respect to the subject probability of the insurance provider.

Next, we observe the large impact of just one loss event on the class of subjective loss probabilities that are compatible with the empirical data. Table 3 reports the upper bound (2) of the proposition in the appendix for the probability of observing exactly one loss event in 100 periods for different values of π. It is obvious that one realisation of the loss state out of 100 periods/samples is not compatible with any π greater than 0.999. This means that one occurrence of a rare event is typically incompatible with beliefs that assign a very low probability to such an event. As a result, one occurrence of a rare event may well result in a substantial revision of beliefs of the agents so that their willingness-to-pay for the insurance rises rapidly since the willingness-to-pay is almost linear in $(1-\pi)$ for small $(1-\pi)$ as the above numerical example illustrates. Note that this is consistent with stylised fact 2, and particularly, stylised fact 3.

Table 3: Probability of Observing Exactly One Loss in 100 Periods

π	Upper bound
0.999999999	2.70468×10^{-7}
0.999999	0.000270441
0.9999	0.026780335
0.999	0.244962197

As noted above, the key observation is that the empirical data for rare events would not be sufficiently large to warrant a robust probability estimate that restricts diversity in terms of insurance premium or willingness-to-pay for insurance. Mathematically, the rate of convergence matters even if (we know that) the strong law of large numbers holds.

3. Welfare Measure under Diverse Beliefs

Let U^h denote the utility of agent h, which may or may not have an expected utility form. It is then standard that a Pareto optimal allocation is characterised as a solution to the social planner's problem, which is based on a social welfare function such that $Z = \sum_{h=1}^{H} \lambda^h U^h$, where λ^h is some positive weight attached to agent h. When U^h has an expected utility form and the beliefs are homogeneous, the above social welfare function can be described as follows:

$$Z = E\sum_{h=1}^{H} \lambda^h u^h = \sum_{h=1}^{H} \lambda^h E u^h$$

since $U^h = Eu^h$ for all h. In contrast, when beliefs are heterogeneous, each utility function U^h is based on a subjective probability.

However, as we explained briefly in the introduction, heterogeneity of beliefs invalidates the standard *ex ante* Pareto optimality and/or social welfare criterion. This is because by allowing for heterogeneous beliefs some agents inevitably hold incorrect beliefs, and such incorrect beliefs cause 'mistakes', which may results in regrets or pleasure *ex post*, even if they act optimally *ex ante* in accord with their beliefs. We use the term 'mistakes', since it is impossible to identify exactly if and how they made 'mistakes' by data when the beliefs are compatible with the data. Ignoring such regrets or pleasure calls for a significant value judgment, since it requires that the inability to hold the correct belief be penalised. In the context of natural disasters, the *ex ante* Pareto optimality requires that any *ex post* relief efforts would typically distort the allocation provided that the insurance policy was available prior to any very rare catastrophes.

Instead of taking such a strong value judgment, and to take *ex post* regrets or pleasure into account, it is probably reasonable to measure the welfare of the agents and the society as a whole with respect to an *ex post* measure. An *ex post* social welfare function is defined by

$$\hat{E}V(u^1, u^2, \dots, u^H)$$

where $\hat{E}$ is the expectation operator with respect to a social probability measure, u^h is the *ex post* utility of agent h (a random variable), and V is a von Neumann-Morgenstern social welfare function, which is a function of the *ex post* utilities of the individuals.

Hammond (1981) shows that a socially optimal allocation based on an *ex post* social welfare function is not Pareto optimal in terms of the *ex ante* expected utilities of the agents unless all agents agree on the probability and the *ex post* social welfare function takes a special form such that

$$\hat{E}V(u^1, u^2, \dots, u^H) = \hat{E}\sum_{h=1}^{H} \lambda^h u^h = \sum_{h=1}^{H} \lambda^h \hat{E}u^h (1)$$

Note that when all agents hold rational expectations, the probability measure that defines $\hat{E}$ is identical to the one in the *ex ante* expected utility function of each agent, and consequently, the *ex post* optimal allocation is identical to the *ex ante* optimal allocation.

Even if we assume that the *ex post* social welfare function takes the form as (1) above, the choice of the social probability measure is not trivial, since there is no way to learn the true probability, and one can only *believe* that his probability belief is the true probability, although one may happen to hold the true probability as his belief. One easy resolution would be to *assume* that the modeller knows the true probability, while the agents in the model don't, and then specify the true probability as the social probability measure. However, such an assumption is not plausible, since apparently no objective justification can be given for the assumption. In other words, *we propose to take a view that the modeller and the agents in the model have equal knowledge and/or ability, rather than taking a paternalistic view that the modeller takes care of the agents in the model by assuming the modeller's possession of superior knowledge and/or ability*.

To follow the principle that the modeller and the agents in the model have equal knowledge and/or ability, the choice of the social probability must be objective, and at least the procedure must be one that can be agreed upon by anyone rational. The rationality requirement here should be a weak one, that the view must not be contradictory to evidence or empirical data. In other words, by taking a frequentist view of probability, and then we may define the acceptable range of subjective probabilities that are compatible with the empirical data. As Nakata, *et al*. (2010) argues and as was explained above, the range of acceptable subjective loss probabilities for rare events would be very large in terms of scale; for instance, 1 in 1,000 and 1 in 1 million are both compatible with the empirical data if the loss event concerned is unprecedented. One way is to define the social loss probability as the average of these acceptable subjective loss probabilities. In doing so, by assuming that the subjective loss probabilities are uniformly distributed, we will be effectively taking the least biased view, since a uniform distribution has the maximum entropy.

Moreover, the lack of knowledge of the subjective probability beliefs of the agents will cause a mechanism design issue, especially when we are to design a decentralised insurance mechanism. This is because we need to be able to predict the behaviour of the agents, which will be influenced by the subjective probability belief. In contrast, the lack of knowledge regarding tastes or degrees of risk aversion would not be too problematic as the above numerical example illustrates.

Suppose there are S states (i.e. the set of all states is defined by $\mathcal{S} \coloneqq \{1, 2, \ldots, S\}$), and let $u^h(w_s^h)$ denote agent$h$'s *ex post* utility in state $s \in \mathcal{S}$ when its wealth level is w_s^h. The *ex post* social welfare is characterised by the utility frontier $\left(\langle u_1^h \rangle_h, \langle u_2^h \rangle_h, \ldots, \langle u_S^h \rangle_h\right)$, where $\langle u_s^h \rangle_h \coloneqq (u_s^1, u_s^2, \ldots, u_s^H)$ for all s. The *ex post* social welfare would be not too far away from the *ex post* social optimum if there is no state s in which w_s^h is extremely low for some agent$h$, regardless of the functional form of u^h. This is a sufficient condition, not a necessary condition, to maintain a reasonably high level of *ex post* social welfare.

Note that this remains the same even if the *ex post* social welfare function does not have an expected utility form (1). Thus, a reasonably high level of *ex post* social welfare is maintained even if preferences of some or all agents do not have a standard expected utility representation and exhibit ambiguity aversion.[5] However, in the case of unawareness or unforeseen contingencies, the set $\mathcal{S}$ itself is unknown.[6] Thus, it is impossible to describe a full state contingent plan, and also the utility frontier $\left(\langle u_1^h \rangle_h, \langle u_2^h \rangle_h, \ldots, \langle u_S^h \rangle_h\right)$ is not well defined.The distinction between unawareness/unforeseen contingencies and the known state space may be important as the empirical study on insurance demand by Nakata, *et al*. (2010) suggests; the willingness-to-pay for flooding insurance is by and large consistent with the subjective expected utility framework, whilst it is not really the case for avian flu insurance, which appears to involve unforeseen contingencies, since mutation is not a foreseen contingency.

5 The Ellsberg paradox by Ellsberg (1962) illustrates the distinction between risk and ambiguity. The latter is formally described by Choquet integral; see Gilboa (1987) and Schmeidler (1989).

6 See Kreps (1979) and Dekel, *et al*. (2001).

4. Comparisons of Regional Insurance Schemes

In this section, we compare several insurance schemes for catastrophes or natural disasters from the viewpoint of the *ex post* social welfare.

4.1. The Macro Risk and the Strong Law of Large Numbers

As stated in the introduction, a typical insurance scheme is based on the strong law of large numbers. Namely, by letting X^h denote the loss of household h, and assuming that X^h is independent for all h,

$$\frac{1}{H}\sum_{h=1}^{H} X^h \to \bar{X} \text{as } H \to \infty,$$

where $\bar{X}$ is some constant. In other words, by expanding the membership of the insurance mechanism $\{1, 2, \dots, H\}$, the average loss per household will converge to a constant with probability one (with respect to a probability measure). In the language of economics, the assumption of independence of X^h across h is stating that each household's loss entirely consists of idiosyncratic risk. However, a catastrophe or a natural disaster typically violates the assumption of independence, which in turn implies the failure of the almost sure convergence of the average loss per household. In other words, a catastrophe incurs some macro risk.

Moreover, the treatment of the idiosyncratic part of the household loss requires great care. This is because it is not obvious at all under what probability measure the strong law of large numbers holds for the idiosyncratic part of the household losses. In other words, the precise properties of the idiosyncratic and the systematic risks of each household's loss are not very straightforward. The diversity of beliefs is inevitable and we need to take into account the impacts of the diversity of beliefs, as the numerical examples in section 2 illustrate.

4.2. Indemnity and Index Insurance

We consider two insurance contracts with different payoff structures: (a) indemnity insurance, and (b) index insurance. A conventional insurance contract is based on the actual loss or the indemnity, and we call the conventional insurance contract as indemnity insurance. Hence, a typical payoff structure of indemnity insurance that covers disaster type(s) k for policyholder i in state s is

$$r_{k,s}^{i} = \min\{\max\{x_{k,s}^{i} - d, 0\}, b\} = \begin{cases} 0 & \text{if } x_{k,s}^{i} \leq d; \\ x_{k,s}^{i} - d & \text{if } x_{k,s}^{i} \in [d, b]; \\ b & \text{otherwise,} \end{cases}$$

where $x_{k,s}^{i}$ is the loss from type k disaster policyholder i incurs, d is the deductible and b is the maximum coverage. Thus, the payoff will exactly match the loss when $d = 0$ and $b \geq \max\{x_{k,s}^{i}, s \in \mathcal{S}\}$. Moreover, the price of the insurance (the insurance premium) is described as $\rho_{k}^{i}(d, b)$, i.e. it is a function of (i, k, d, b). For standardised indemnity insurance, the premium is not exactly personal to the policyholder i, but is a function of the attributes of i. In such a case, $\rho_{k}(\theta, d, b)$, where θ is the attributes of the policyholder.

Meanwhile, index insurance is a contract whose payment is contingent on a set of pre-determined conditions, and is not based on the actual loss the policyholder incurs. Usually the pre-determined conditions (i.e. the trigger event) are easily observable and/or verifiable. For instance, a pre-determined amount (e.g. USD 1 million) of an earthquake index insurance contract will be paid to the policyholder if an earthquake that is at least as powerful as the specified level (e.g. magnitude 7.0) occurs in a specific location (e.g. the epicentre is within 100 miles from Tokyo’s city centre). Thus, the payoff structure of a typical index insurance contract for the trigger event k (i.e. the set of states $\mathcal{S}^{k} \subset \mathcal{S}$) is

$$r_{k,s} = \begin{cases} c & \text{if } s \in \mathcal{S}^{k}; \\ 0 & \text{otherwise.} \end{cases}$$

Note that the payoff $r_{k,s}$ is not a function of i, i.e. it is the same across the policyholders. There will be a discrepancy between the payment of the index insurance and the actual loss from the perspective of the insurance policyholder, and such discrepancy is often referred to as a basis risk in this context.

Each index insurance k can be described as a zero-net supply security:

$$\sum_{i \in \mathcal{I}} z_k^i = 0,$$

where $\mathcal{I}$ is the set of all economic agents (households, firms, local governments, etc.) and z_k^i is economic agent i's position of index insurance k. Indeed, the market structure of the index insurance can be either (a) a competitive market along the line of a standard general equilibrium model, where all economic agents are price takers, or (b) a typical market for insurance products, where insurance companies control prices and hold short positions. Practically, the former case would require that the index insurance to be traded on the capital market, just like the catastrophe (CAT) bonds. In the latter case, we may fix the identity of the supplier of each index insurance k, i.e. we can simply add conditions such that $z_k^i < 0$ for the particular supplier i.

In the former case, the price q^k satisfies the following equation in equilibrium:

$$q^k = \sum_{s \in \mathcal{S}} \pi_s^i \frac{\partial u^i(w_s^i)}{\partial w_s^i} r_{k,s},$$

where π_s^i is economic agent i's subjective probability of state s, w_s^i is agent i's wealth in state s, and u^i is agent i's (indirect) utility function. This equation states that the agent may choose to allow for the occurrence of a state with a very low w_s^i when π_s^i is very small. In other words, agents that underestimate the catastrophe states would limit the purchase of index insurances that could compensate for the losses in those states.

It is clear that both types of insurance contracts are state contingent claims. Thus, the verification of the state is crucial. However, verification is much costlier and is more prone to moral hazard for the indemnity insurance than for the index insurance, since the latter only requires the verification of $s \in \mathcal{S}$, whilst the former requires the verification of $x_{k,s}^{i}$.

4.3. Evaluations of Different Insurance Schemes

Next, we attempt to compare and evaluate the indemnity and index insurance. In so doing, we consider three different cases concerning the identity of the policyholders: (a) households, (b) local governments, and (c) national governments. The first case is the most obvious one, in which each household directly purchases insurance policies. The second and the third are the cases in which the insurance policies are purchased by the governments (local or national), and the purchases may be financed by tax.

Let N denote the set of countries (that participate in the regional insurance scheme), and n its typical element, i.e. country n. Let $\mathcal{L}$ denote the set of all local governments in the region, and l its typical element, i.e. local government l. If necessary, we let $\mathcal{L}^n$ denote the set of all local governments in country n. We may let $\mathcal{H}^l$ denote the set of all households in the area of the local government l.

4.3.1. Direct subscriptions by Households

We first consider the case in which each household is the potential policyholder of the insurance, either indemnity or index insurance. This is desirable with respect to the *ex ante* Pareto optimality criterion, since the choice made by each household will reflect its *ex ante* preference.

However, as we showed above, the subjective loss probability would be rather diverse especially for households who have experienced no major losses in the past. When the insurance premium is set by the supplier, this would result in a rather low level of subscription rate, since a large number of households would deem the premium too high. Also, for index insurance, when the market structure is competitive (i.e. all economic agents are price takers), households whose probability estimates for the catastrophe states are very low would not hold a position that would sufficiently cover the losses in the catastrophe states. In these cases, the *ex post* welfare of the economy would become very low, since some households would be left in a disastrous condition, if no relief efforts are made *ex post*.

4.3.2. Subscriptions by Local Governments

Next, we examine the case in which local governments are the potential policyholders of the insurance, instead of the households. For every local government l, the aggregate loss of the households in the governing area is $X^l \coloneqq \sum_{h\in\mathcal{H}^l} X^h$. When there is no macro risk at each local government level, this quantity is a constant. However, there typically remains a macro risk at this level especially for natural disasters. Thus, there are incentives for the local governments to share risk with other local governments within the country or within the whole East Asian region.

Moreover, to finance the insurance premium, the local government would either use its general tax income or impose a separate tax specific to the insurance. For index insurances, it is possible conceptually that the cost of the portfolio of index insurances may be zero, $\sum_{k\in\mathcal{K}} q^k z_k^i = 0$, where $\mathcal{K}$is the set of all index insurances. Either way, this scheme is effectively a two-tier risk sharing scheme. That is, (a) the risk sharing scheme amongst local governments, and (b) the risk sharing scheme within the governing area of each local government. The latter scheme should be designed so that it eliminates idiosyncratic risk at the household level.

One major advantage of this scheme over the one with direct subscriptions by households is that the subjective probability of X^l would be less diverse than that of X^h for household h with no or very limited prior loss experience, since the empirical loss probability of $X^l$is larger than that of X^h for households with no or very little prior loss experience. Thus, the conflict between the *ex ante* and *ex post* welfare measure would be less severe, at least in terms of decision making. However, many households with no prior loss experience may well view the scheme *ex ante* as a wealth transfer mechanism that would be disadvantageous to them.

The implementation of the risk sharing scheme within the governing area of each local government may be very costly and prone to moral hazard if it follows the design of indemnity insurance, i.e. the payments to the households are made against the claims made by the households. However, if the payments are made so that no household would be left in a devastating state even if they are not exactly matching the actual losses, the *ex post* welfare would be reasonably high.

4.3.3. Subscriptions by National Governments

Finally, we examine the case in which national governments are the potential policyholders of the insurance. For every national government n, the aggregate loss of the households in the governing area is $X^n := \sum_{h \in \mathcal{H}^n} X^h$. Clearly, the subjective probability of X^n would be less diverse than that of X^l for all $l \in \mathcal{L}^n$. In this case, the regional insurance scheme aims at sharing the macro risks at the country level, whilst the risk sharing within the country will be done through the tax system that would finance the regional insurance premium.

One major problem is that the determination of the insurance premium would be very political, as we observe in many international frameworks. This applies also to index insurance, since no national government would act as a price taker.

5. Conclusion

We have examined the possible issues that are key to design regional insurance schemes for catastrophes or natural disasters that mainly target the household sector. We first introduced a simplified version of the insurance demand model by Nakata, *et al*. (2010), which is consistent with the three stylised facts about catastrophe insurance demand. The key observation is that the robustness of a probability estimate of a rare event is very limited, which would result in a large diversity and variability in the scale of subjective loss probabilities for rare events.

When the probability beliefs are diverse, the standard Pareto criterion becomes dubious, because it is based on the *ex ante* preferences, which govern the decisions of the agents, but ignore the regrets for the mistakes made due to the 'incorrect' beliefs. Thus, it would be sensible to use the *ex post* welfare measure proposed by Starr (1973) and Hammond (1981). However, the choice of social probability for the *ex post* social welfare function with an expected utility form is not straightforward. Nevertheless, by making everyone avoid any catastrophe state would ensure that the *ex post* social welfare would be close to the *ex post* social optimum, regardless of the true social probability or the functional form of the *ex post* utility.

With this in mind, we evaluated various insurance schemes. For both the indemnity and index insurance schemes, voluntary direct subscriptions by the households are not desirable, since voluntary subscriptions by the households would most likely to lead to insufficient level of insurance coverage and the occurrence of personal catastrophe states for some agents due to the large diversity in subjective loss probabilities. Since the diversity in the loss probabilities would be less for aggregate losses at the local government level, an insurance scheme with subscriptions by local governments in conjunction with *ex post* payments/compensations to the affected households would be more desirable. Considering the possible moral hazard issues inherent to indemnity insurance, schemes based on index insurance appear to be more desirable. However, the underwriting costs for index insurance may well not be low, whether the index insurance will be supplied and priced by insurance suppliers or traded on the capital market.

The current paper leaves several important issues unexamined. First, supply side issues, including but not limited to the issues related to underlying costs, are not examined, and they require both empirical and theoretical examinations. Moreover, analyses based on a dynamical model would be needed. As noted above, catastrophes or natural disasters tend to incur risk at the aggregate level (i.e. macro risk). Thus, it is impossible to exactly match the insurance payment of indemnity insurance with fixed insurance premiums in every period. Hence, a dynamical model is needed to analyse the level of reserves needed to ensure smoothing of aggregate wealth or consumption over time, without falling to insolvency. Also, for index insurances traded on the capital market, the impacts of possible fluctuations in (relative) prices should be analysed by a dynamical model, since the fluctuations may well have significant impacts.

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Appendix: The large deviation property

In what follows, we reproduce the exposition of Lemma 1.1.9 of Dembo & Zeitouni (1998) in Nakata, *et al*. (2010). Let random variable X_t denote the loss in period t, and let $X_1, X_2, \dots, X_T$ be an i.i.d. sequence. Also, let $\mathcal{P}(\mathcal{A})$ denote the space of all probability laws on $\mathcal{A} \coloneqq \{a_1, a_2, \dots, a_S\}$. Furthermore, for a finite sequence (of realisations) $\mathbf{x}^T = (x_1, x_2, \dots, x_T)$, we define the empirical measure of a_s as follows:

$$m_T^{\mathbf{x}}(a_s) \coloneqq \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{a_s}(x_t), \quad \forall s,$$

where $\mathbf{1}_{a_s}(\cdot)$ is an indicator function such that

$$\mathbf{1}_{a_s}(x_t) = \begin{cases} 1 & \text{if } \ x_t = a_s; \\ 0 & \text{otherwise.} \end{cases}$$

Then, we define type $m_T^{\mathbf{x}}$ of $\mathbf{x}^T$ as

$$m_T^{\mathbf{x}} \coloneqq \left(m_T^{\mathbf{x}}(a_1), m_T^{\mathbf{x}}(a_2), \dots, m_T^{\mathbf{x}}(a_S)\right).$$

Let $\mathcal{M}_T$ denote the set of all possible types of sequences of length T, i.e.

$$\mathcal{M}_T \coloneqq \{\nu : \nu = m_T^{\mathbf{x}} \text{ for some } \mathbf{x}^T\}.$$

Also, the empirical measure $m_T^{\mathbf{x}}$ associated with a sequence of random variables $\mathbf{X}^T \coloneqq (X_1, X_2, \dots, X_T)$ is a random element of $\mathcal{M}_T$.

Let P_π denote the probability law associated with an infinite sequence of i.i.d. random variables $\mathbf{X} = (X_1, X_2, \dots)$ distributed following $\pi \in \mathcal{P}(\mathcal{A})$. Also, the relative entropy of probability vector ν with respect to another probability vector π is $H(\nu|\pi) \coloneqq \sum_{s=1}^{S} \nu_s \ln \frac{\nu_s}{\pi_s}$.

Proposition (Lemma 1.1.9; Dembo and Zeitouni, 1998): *For any* $\nu \in \mathcal{M}_T$,

$$(T+1)^{-S} e^{-TH(\nu|\pi)} \leq P_\pi(m_T^{\mathbf{x}} = \nu) \leq e^{-TH(\nu|\pi)} \qquad (2)$$

The proposition states that the probability of observing type ν for a sequence of length T with respect to probability law π^{π} has the lower and upper bounds as specified in (2).[7] Clearly, both the lower and upper bounds are decreasing in $H(\nu|\pi)$. Note that this result (and the results in the literature of large deviations) is very useful, since it may well be rather difficult to compute the exact probability $P_\pi(m_T^{\mathbf{x}} = \nu)$ in many cases. This difficulty arises from the fact that we need to consider all possible paths/sequences that belong to the specified type, which involves combinatorics. Moreover, from this result, we know that the relative entropy $H(\nu|\pi)$ characterises the probability $P_\pi(m_T^{\mathbf{x}} = \nu)$, although the bounds may not be very tight in some cases.

[7] $P_\pi(m_T^{\mathbf{x}} = \nu)$ is a likelihood function in the language of Bayesian statistics, in which case an explicit updating of beliefs is modelled. However, we do not assume such an explicit belief updating mechanism in the current paper.