

# Chapter 3

## Measure the Energy Market Integration in East Asia: A Principal Component Analysis Approach

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## CHAPTER 3

# Measure the Energy Market Integration in East Asia: A Principal Component Analysis Approach<sup>1</sup>

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### ABSTRACT

*This paper measures the current energy market integration (EMI) in the 16 East Asia countries, comprising the ASEAN 10 countries, China, Japan, Korea, India, Australia, and New Zealand, by using the principal component analysis (PCA) approach. This comprehensive EMI index has four important components: (1) energy trade liberalization; (2) energy infrastructure development; (3) energy market liberalization; and (4) energy pricing liberalization. This index is constructed in two steps. I first construct the four indicators using PCA. After the predicted observation for the four indicators are obtained, I once again adopt the PCA method to calculate the EMI index. The scores show that countries like Japan and New Zealand have the highest extent of energy market integration. In contrast, countries like China and Malaysia, and India have lowest scores of EMI. Poorer countries are located in between. Such results are robust to different measures or data adopted.*

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## 1. Introduction

In East Asia Summit region, ASEAN has long been pursuing the energy market integration (EMI) to support their sustainable economic growth. The first energy agreement, concluded between Thailand and Lao PDR, was signed in 1966, one year before the first ASEAN Declaration in August 1967. After the establishment of the ASEAN Council on Petroleum (ASCOPE) in 1975, cooperation widened to include all other fuels. In 1981 the Heads of ASEAN Power Utility Authorities (HAPUA) was established for work on electricity interconnection, and in 1986 the ASEAN Energy Cooperation Agreement outlined a wide range of areas for cooperation.

The great efforts that member countries have made in the past four decades has lead to significant progress in the direction of forming a regional unified energy market. To further promote integration, more information on the status of each country's extent of integration should be measured to inform the corresponding government for their future policymaking.

ASEAN 10 countries, China, and India, the so-called ACI countries, are still net importers of energy products such as oil, coal, natural gas, liquid national gas (LNG), and electricity from the rest of the world. Although the energy in Asia as a whole is almost self-sufficient, the energy supply is imbalanced between different regions. Due to the fast economic growth in the ACI bloc, such countries are experiencing a strong energy demand today. Studies like *World Energy Outlook* (2009) predict that the ACI countries will remain as trailblazers with respect to projected growth in primary energy demand. In particular, the annual energy demand of India will grow at 3.4 percent, followed by China at 2.9 percent, and ASEAN countries by 2.5 percent during 2010-2030. Given this growing demand of energy in East Asia area, there is an urgent need for such countries to join together to work for a regional energy integrated market.

According to recent work by Shi and Kimura (2010), the next steps for further EMI in the region lie in three areas: (1) regional agreements on energy trade and investment; (2) energy infrastructure development and national energy market liberalization; and (3) energy pricing reform and fossil fuel subsidies. Due to disparities in the level of economic development across countries, each country will have different abilities to participate in each dimension.

To assist policy making, this study aims to build up an index system by using the principal component analysis approach. The analysis measures the status of each country in EMI process of East Asia Summit region without imposing weights for each dimension. Contributing to previous literature, the study not only provides the aggregate level measure of EMI, but also information on each dimension that is comparable across countries, so that priorities for next-step in EMI can be identified.

The rest of the paper is organized as follows. Section 2 introduces some stylized facts on current situation of energy market integration in the ASEAN+6 countries, followed by an introduction of the principal component analysis in Section 3. Section 4 examines the predicted score of all sub-indicators for energy integration in this area, and accordingly, the final score for each country. In Section 5, the indexes are tested for robustness. Section 6 concludes.

## **2. Energy Market Integration in East Asia**

As recognized by Feenstra (1998), trade integration and production disintegration are the two important features of international trade today. Like other manufacturing industries, this is true for energy trade as well. In this section, I investigate energy supply in East Asia area. To better understand regional trade, I also explore current energy demand in East Asia. My finding is that East Asia is an energy-thirsty region, despite its relatively abundant energy resources. Accordingly, regional energy trade plays an important role for sustainable growth in East Asia area. Energy market integration in East Asia remains needed and urgent.

### **2.1. Energy Supply in East Asia**

East Asia is a relatively resource-rich area in terms of both energy reserves and current available supply. Located in southeast of Asia, most ASEAN+6 countries have substantial energy resources. As shown in Table1, eight of the sixteen countries have proven oil and gas reserves, and seven countries have substantial coal reserves. Moreover, China and the Northern part of the ASEAN region are rich in enormously

powerful stream power which can be harnessed to generate electricity (Nicolas, 2009).

Oil production in East Asia is sizable. In particular, China is the 5th largest oil-producing countries in the world, according to the newest *BP statistical review of World Energy (2010)*. India, Malaysia and Indonesia also produce large amounts of oil. Turning to the reserves and production in the gas market, one sees a similar story. ASEAN+6 countries hold more than 7% of global proven natural gas reserves, with the most significant reserves in Indonesia, China and Malaysia. East Asia accounts for around 12% of world natural gas production.<sup>3</sup>

In addition, this region holds considerable amounts of coal, especially in China and India. Since most of ASEAN+6 are developing countries, coal still plays a vital role. Today it is already widely recognized that East Asia has been the most important “world factory” and enjoyed the fast economic growth. Yet, without the abundant resources of coal, it is difficult to imagine countries in the area to achieve such economic growth.

**Table 1. Energy Resources in East Asia (Reserves and Production) in 2009**

Types	Oil		Natural Gas		Coal	
	Reserves (thousand million tons)	Production (million tons)	Reserves (trillion cubic meters)	Production (billion cubic meters)	Reserves (million tons)	Production (MTOE)
Brunei	0.1	8.2	0.35	11.4	—	—
China	2.0	189.0	2.46	85.2	114500	1552.9
India	0.8	35.4	1.12	39.3	58600	211.5
Indonesia	0.6	49.0	3.18	71.9	4328	155.3
Malaysia	0.7	33.2	2.38	62.7	—	—
Myanmar	—	—	0.57	11.5	—	—
Thailand	0.1	13.6	0.36	30.9	1354	5.3
Vietnam	0.6	16.8	0.68	8.0	150	25.2
Japan	—	—	—	—	355	0.7
South Korea	—	—	—	—	133	1.1
World Total	181.7	3820.5	187.49	2987.0	826001	3408.6

Source: BP Statistical review of World Energy 2010.

Although the overall energy resource availability is not a major challenge in East Asia, resource allocation is uneven in this region. Some countries are energy resource

<sup>3</sup> This is comparable to all America's reserves (BP Statistical review of World Energy 2010 )

abundant whereas some others are resource scarce. As shown in Table 1, some of the northern East Asian countries like Japan and South Korea have close to no energy resources, despite being the most developed countries in East Asia. In sharp contrast, China and India, as the two emerging giants, are relatively abundant in energy reserve and production. Another notable exception is Singapore, which is deprived of any energy natural resources and accordingly heavily depends on its immediate neighbors (i.e., Indonesia and Malaysia) for its energy supply (Nicolas, 2009).

The third characteristic of energy situation in East Asian countries is the strong growing demand. In particular, according to the prediction of *World Energy Outlook* (2009), ASEAN primary energy demand expands by 76% between 2007 and 2030, an average annual rate of growth of 2.5%. This is much faster than the average rate in the rest of the world. Annual energy demand of India is expected to grow at 3.4 percent, followed by China at 2.9 percent. Whilst still in safe greenhouse gas (GHG) emission levels, the annual demand growth of ASEAN (2.1%) is still much higher than global average (1.5%). Take China for example, it will overtake the United States to become the world's biggest importer of oil and gas within a decade. China, along with India, also has the highest expected growth rate of gas consumption in the first three decades of the new century. All of these statistics clearly suggest that East Asia is a substantial energy market in the world today.

In summary, although East Asia as a whole is a relatively resource-abundant region, its energy market is imbalanced. Energy supply cannot meet the the rapid increase in primary energy demand. Such an excess demand for the whole area calls for further efforts in regional cooperation and intra-regional energy trade. The issue of energy market integration in East Asia is still ongoing, as discussed in the following subsection.

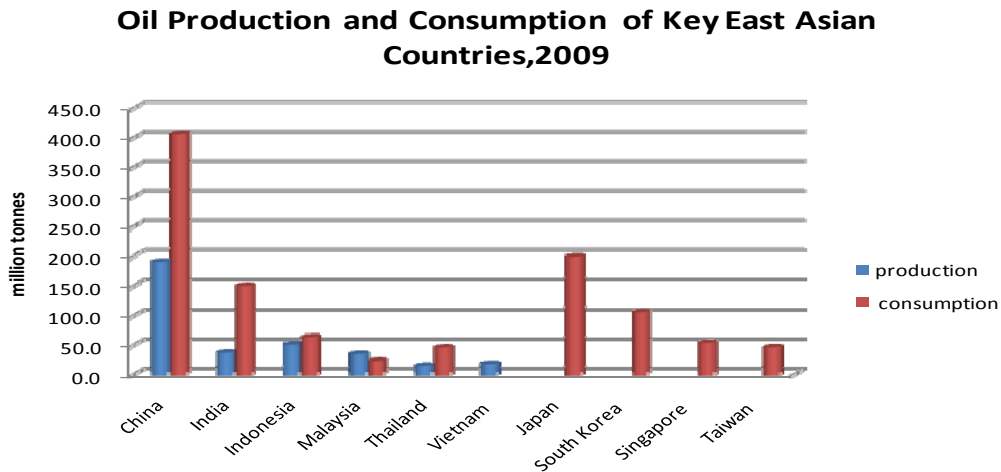
## **2.2. Energy Trade within East Asia**

As mentioned above, Asia as a whole is nearly energy self-sufficient. East Asia holds vast oil, coal, and natural gas resources. However, the area is still a net importer of oil due in large part to its uneven resource allocation and high growth in energy demand. This uneven allocation presents an urgent need for regional energy trade. In this subsection I will investigate the current situation of energy trade in the area by each energy type: oil, natural gas, coal, and electricity.

### 2.2.1. Oil Trade

We first investigate the oil trade. As shown in Figure 1, there is a large gap between oil production and consumption of East Asian countries in 2009. Among these countries, China, India, Japan, and South Korea are the largest importers. The total oil imports of China, India, and Japan reached 612 million tons in 2008. Such a number is close to the oil imports of either US or Europe, amounting respectively to 637 and 681 million tons. By exporting its imported oil, Singapore indeed becomes the largest exporter and largest entrepôt in this region. Other oil exporters in East Asia include countries like Australia, China, India and Japan, but their export volumes are small.

**Figure 1. Oil Production and Consumption for Key East Asian Countries**

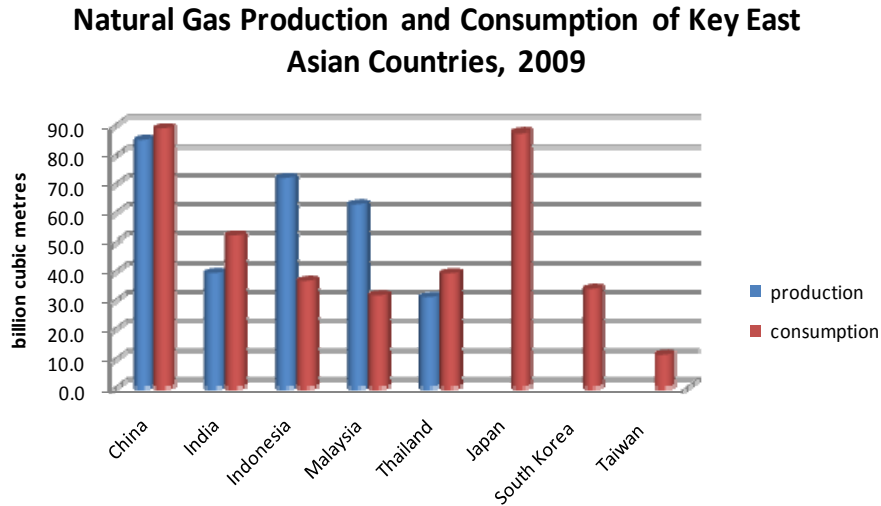


Source: BP Statistical review of World Energy 2010.

### 2.2.2. Natural Gas Trade

Natural gas is traded by pipeline or shipped as liquefied natural gas (LNG). As shown in Figure 2, China, Thailand and India have a small excess demand. Other countries like Japan, South Korea, and Taiwan have much greater excess demand. Indonesia and Malaysia have extra large supplies available for export.

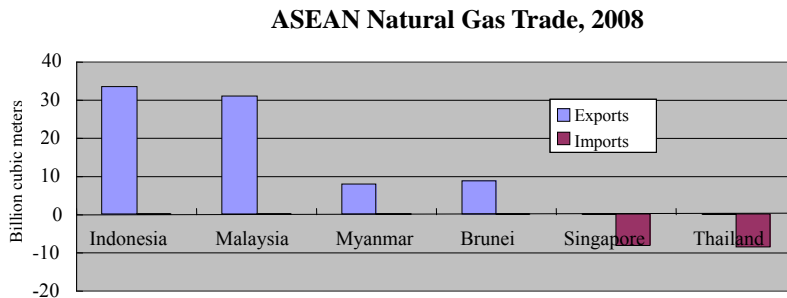
**Figure 2. Natural Gas Production and Consumption in the Key East Asian Countries**



Source: BP Statistical review of World Energy 2010.

Figure 3 hence shows the natural gas trade pattern by pipeline of ASEAN countries. Without a surprise, countries like Indonesia, Malaysia, Myanmar, and Brunei are net exporters of natural gas whereas Singapore and Thailand are net importers. In particular, Indonesia, both Malaysia, and Myanmar export natural gas to a single importer only. Precisely, Myanmar exports only to Thailand; and Indonesia and Malaysia export only to Singapore.

**Figure 3. Natural Gas Traded by Pipeline in ASEAN Countries**

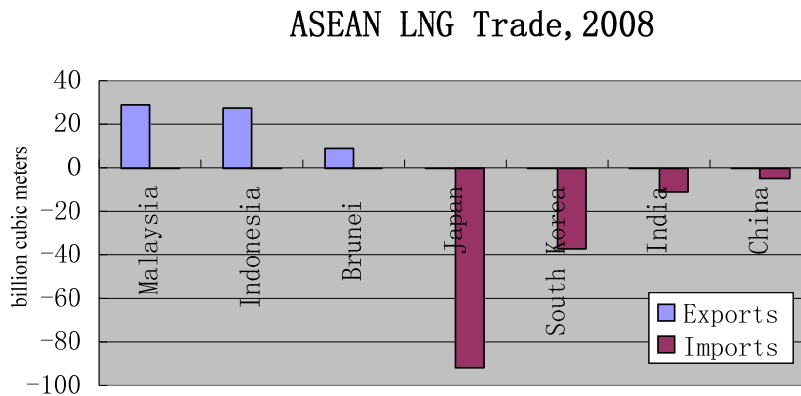


Source: BP Statistical review of World Energy 2009.



Natural gas is also shipped as LNG in East Asia. Once again, the main LNG exporters are Malaysia, Indonesia and Brunei. Turning to the importer's side, Japan's LNG imports alone represented more than 40% of the world's total. In addition, countries like South Korea, India, and China, are other large LNG importers in Asia, as presented in Figure 4.

**Figure 4. Natural Gas Traded as LNG in ASEAN Countries**



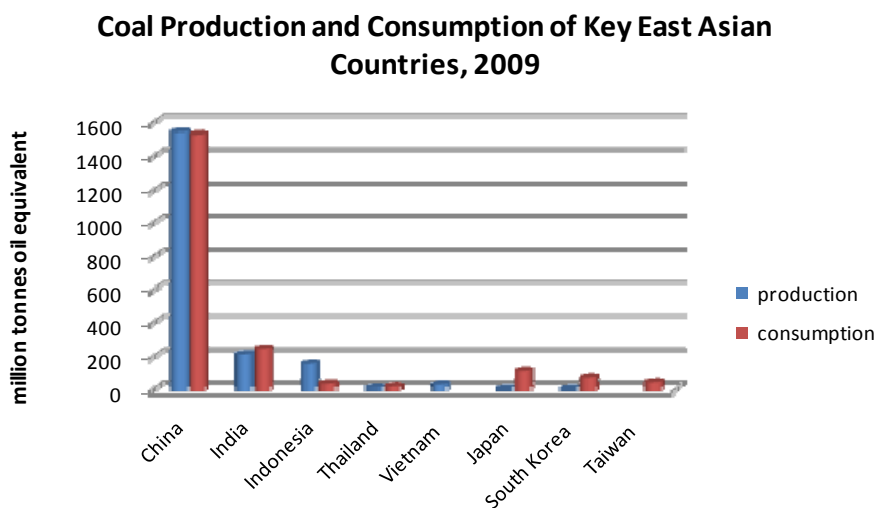
Source: BP Statistical review of World Energy 2008.

### 2.2.3. Coal Trade

It is well known that China is a large supplier of coal, producing 1552.9 million tons oil equivalent in 2009. China is also a huge coal consumer due to its fast economic growth. In 2009 its consumption on coal reached 1537.4 million tons. Both these two statistics dwarf their counterparts for other countries in East Asia, as shown in Figure 5. Coal exports from China, however, was declining at the rate of more than 12% per year in 2004-2007, even though coal production was growing at more than 8% during the same period. This was due in large part, to a growing domestic demand in China. Turning to other countries in the region, India ranks No. 2 in terms of both production and consumption of coal, though the magnitudes are only around 1/8 of China's production and consumption. The largest exporters in this market are Australia and Indonesia. Their total exports are around 491 million tons of coal exports in 2007, which represented 46% of the global exports. China, India,

Japan and South Korea are the coal importers in the region.

**Figure 5. Coal Production and Consumption of Key East Asian Countries**



Source: BP Statistical review of World Energy 2010 .

#### 2.2.4. Electricity Trade

China is the largest exporter of electricity in East Asia area and exported 14 TWh of electricity to Hong Kong and Macau in 2007, representing 19% of electricity exports in the whole Asia. Simultaneously, China imported 4.77 TWh, which represented 8% of electricity imports in the whole Asia. Besides of these, there are three completed electricity interconnections that facilitate electricity trade in the Southeast Asia sub-region. Namely, the Thailand- Malaysia market, Malaysia-Singapore market, and Thailand-Lao PDR market. In particular, India and Thailand are the two important importers of electricity in the region, respectively importing 4.96 TWh and 4.488 TWh in 2007.

### 2.3. Energy Market Integration in East Asia Today

Countries in East Asia have long made efforts to make integrate their energy markets integration. Perhaps the first effort is the energy agreement signed by Thailand and the Lao PDR in 1966, one year before the first ASEAN Declaration. After that, cooperation in all fuels of various forms has been gradually planned and

achieved. Shi and Kimura (2010) provide a nice summary on past and current efforts made to foster energy market cooperation. EMI in East Asia was conducted by the following three components: (a) a series of ASEAN Plans of Action for Energy Cooperation (APAEC, 1999, 2004, 2009) which aims to highlight the importance to construct a reliable, transparent, and cooperative energy market; (b) the ASEAN Economic Community (AEC) blueprint emphasizes the establishment of interconnecting arrangements through regional cooperation in Trans-ASEAN Energy Networks comprising the Trans-ASEAN Gas Pipeline (TAGP) and the ASEAN Power Grid (APG) (APAEC, 1999); and (c) financial support for energy market cooperation. ASEAN receives large amount of funds for programs on coal and clean coal technology, energy efficiency and conservation (EE&C), renewable energy and regional energy policy and planning from dialogue partners, namely, the European Union, Japan, Australia, China, Korea, and India.

The regular meetings of ministers in East Asian countries and particularly ASEAN countries play a vital role in fostering regional EMI. Beyond ASEAN, many institutional cooperation frameworks have emerged in East Asia under the principle of ASEAN centrality in the past decades, such as ASEAN Plus One, ASEAN Plus Three (ASEAN plus China, Japan, and Korea) and EAS. There are also regular energy Ministers' meetings under these frameworks. Many work plans and programs have been adopted in the meetings, on fields such as energy security, oil markets, renewable energy and energy efficiency and conservation.

Table 2 presents the most important features of the energy market integration in Shi and Kimura (2010).

**Table 2. Overview of EMI current status in Shi and Kimura's study (2010)**

Areas	Current Status
Regional agreements on energy trade and investment	<p><b>Trade and investment has been broadly covered in the existing Bilateral/multilateral free trade agreement.</b></p> <p>Beyond its own area, ASEAN has conducted negotiations for free trade (FTAs) and comprehensive economic partnership agreements (CEPAs) with many dialogue partners, including the "plus six" countries</p> <p>Bilateral FTAs between individual ASEAN member country and ASEAN dialogue countries have been moved forward while bilateral FTAs among the ASEAN dialogue partners are largely under negotiation.</p>
Energy infrastructure development	<p><b>Currently proposed energy infrastructure projects have been limited to the ASEAN +China region only, though India has the potential to link.</b></p> <p>Two flagship projects are APG and TAGP. Political trust is a huge barrier to trade in pipeline gas and electricity, and thus the demand for energy infrastructure.</p>
National Energy Market Liberalization	<p><b>Market liberalization has been attempted in some countries, but lot more remains to be done.</b></p> <p>In the EAS region, energy market liberalization has been conducted in Australia, Japan, New Zealand, the Philippines, and Singapore, while in others, energy markets are more or less restricted in some of the following ways: markets are dominated by some vertically integrated suppliers, prices are regulated, trade qualification is limited, electricity networks/gas pipelines are not open to access, and so on.</p>
Energy pricing reform and fossil fuel subsidies	<p><b>Price restrictions and subsidies for energy commodities are often used in many EAS countries.</b></p> <p>Energy prices have been liberalized in Australia, Japan, ROK, New Zealand, and the Philippines. Prices of electricity are more often regulated than coal, oil and natural gas.</p>

Source: Shi and Kimura (2010).

The current consensus is that the further energy market integration is not only a requirement for the regional economics, but also good for the increasing the wealth of people in East Asia. Voluntary integration of energy markets will need to respect the

different contexts facing each nation, and it is imperative to establish the current extent of the EMI. Without such information, there is no direction for policy makers. This analysis provides a reliable quantitative index to establish each nations' current degree of EMI, by using principal component analysis. I now provide an overview into the methods used in principal components analysis to construct an index.

### **3. The Method of Principal Component Analysis**

Principal component analysis (PCA) is a way of identifying patterns in data, and expressing data in such a way as to highlight their similarities and differences. Since patterns in data can be hard to find in data of high dimension, since the luxury of graphical representation is not available, PCA is a powerful tool for analyzing data so as to form a comparable index across countries under the condition that there is no explicit weight available for the various components.

The PCA approach is an ideal instrument to explore the energy market integration in East Asia for two reasons. First, it is ideal to use a comprehensive indicator to measure energy market integration since any particular indicator can only interpret one perspective of the energy market integration in this emerging economy, which would lead to a possible measurement bias. Second, given that multiple indicators to measure the EMI is a must, any arbitrary weight among such indicators would create another serious estimation bias. By contrast, an adoption of the PCA approach can deal with such two empirical challenges well. Appendix A provides a careful scrutiny of the detailed technique of the PCA approach adopted in the present paper.

### **4. Measuring Energy Market Integration**

The main aim of this section is to measure and calculate the scores of energy market integration for the ASEAN+6 countries. The final score comes from the main components of the following four indicators: (1) regional energy trade liberalization; (2)

energy infrastructure development; (3) national energy market liberalization; and (4) energy pricing liberalization, as suggested by Shi and Kimura (2010).

Two steps are required to calculate the final integration score. First, I determine the sub-components for each main component. Once the sub-components are chosen, I can adopt the PCA method to calculate its predicted principal component as an index. Second, with these four predicted indicators at hand, I am able to calculate the predicted principal component (i.e., the final score) of the market integration by country.

The rest of this section is organized as follows. I first examine each sub-component for each main indicator to obtain its predicted score, followed by the calculation of the final aggregated score of energy market integration by country.

#### **4.1. Energy Trade Liberalization**

As documented in Nicolas (2009), the ASEAN Free Trade Agreements were launched in 1992. The full free trade is set for 6 original ASEAN countries in 2010, and free trade expands before 2015 for the CMLV (Cambodia, Myanmar, Laos, and Vietnam) groups. The current objective for ASEAN countries is to form a larger FTA, including other Asia-Pacific countries. There are three different schedules widely discussed. The first possibility is that the ASEAN countries join with China to have a new ASEAN+1 free trade area. The second possibility is to an ASEAN+3 FTA to include both ASEAN 10 countries and three other countries: China, Japan, and Korea. The last possibility is to extend the ASEAN+3 FTA to ASEAN+6 by including Australia, India, and New Zealand. The present paper takes a broader view, following the last suggestion to examine energy market integration in the 16 Asia-Pacific countries.

The coverage of the current AFTA not only includes the regular tariff reduction on commodities but also the phasing-out of various non-tariff barriers. In particular, the AFTA has a focus on the free-trade oriented energy sector. Currently the ASEAN 10 countries have already created a FTA with some other countries in the Asia-Pacific area. Therefore, to measure the trade liberalization in the energy sector for each country, we use the number of countries that have a FTA relationship with the country in the Asia-Pacific area. For example, each ASEAN country has already signed or completed the FTA agreements with Australia, China, India, Japan, Korea, and New Zealand, but it is not true in other cases. China only signed the FTA with ASEAN 10 countries and

New Zealand, and is negotiating with Australia for a possible FTA. In this way, each ASEAN country has a score with 16 to count number of countries with FTA agreement. In contrast, China is only assigned with 12 given that it currently has no view to sign a FTA with the other four countries.

**Table 3. Status of FTA/EPAs in the EAS Region**

	Australia	China	India	Japan	New Zealand	South Korea	ASEAN
Australia		□		□	●	□	●
China	□				●		●
India				□		□	●
Japan	□		□			□	●
New Zealand	●	●				□	●
South Korea	□		□	□	□		●
ASEAN	●	●	●	●	●	●	

*NOTE:* ●: FTA signed/concluded; □: under negotiation

*Source:* Shi and Kimura (2010).

However, this is far from the whole story of energy trade liberalization. There still exist some other economic indicators to help us understand energy trade liberalization in the East Asia region. Here I consider the following five indicators: (1) the ratio of energy net imports over consumption. A large number of this index indicates that the country strongly depends on international energy markets, since most of its domestic energy consumption is imported. (2) Energy production (thousand tons of oil equivalents) and total energy consumption. These two indicators measure the economic size of the energy market in a country. (3) Per-capita energy consumption (kg of oil equivalent per capita) which captures both the economic size and population of a country. (4) GDP per unit of energy use (constant 2005 PPP \$ per kg of oil equivalent), which assess the efficiency level of energy use. The larger the number, the more efficient the country is. Table 4A describes the basic summary statistics for the indicators above.

**Table 4A. Summary Statistics for Indicators of Energy Trade Liberalization**

Variable	Mean	Std. Dev.	Min	Max
Number of FTA Agreements Signed	9.625	0.957	9	12
Ratio of Energy Net Imports over Consumption	-35.13	170.3	-630	100
Energy Production (kt of oil equivalent)	221997	445529	0	1.80E+06
Total Energy Consumption (kt of oil equivalent)	262702	485155	2767	2.00E+06
Per-capita Energy Consumption (kg of oil equivalent)	2694	2285	319	7190
GDP per unit of energy use	5.876	1.455	4	9

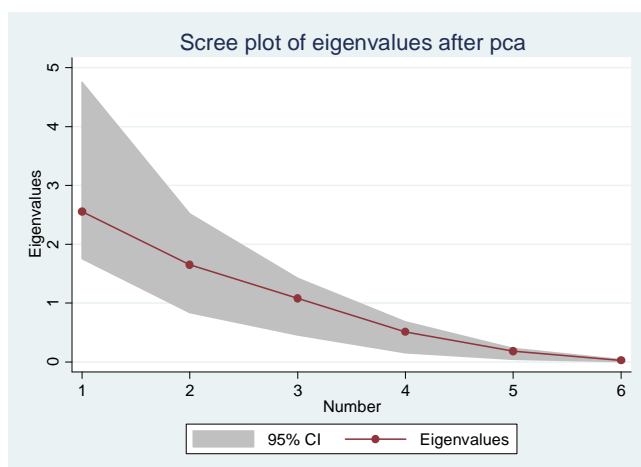
Based on the above information, I am able to calculate the predicted score for the aggregated energy trade liberalization. For the sake of completeness, table 4B reports and sorts the Eigenvalues of six eigenvectors associated with data on energy trade liberalization.

**Table 4B. The Eigenvalue for the PCA for the Energy Trade Liberalization**

Component	Eigenvalue	Difference	Proportion	Cumulative
C1	2.5529	0.9009	0.4255	0.4255
C2	1.6520	0.5710	0.2753	0.7008
C3	1.081	0.5741	0.1802	0.881
C4	0.5069	0.3253	0.0845	0.9655
C5	0.1816	0.1560	0.0303	0.9957
C6	0.0256	.	0.0043	1

Source: Author's own calculation.

Figure 2 sorts the six eigenvalues from top to down and report its 95% interval confidence. Here I also compute heteroskedastic bootstrap confidence intervals at the 95% level. Clearly, the highest eigenvalue is 2.552 whereas the lowest one is .02.

**Figure 2. Plots of the Eigenvalues for the Index of Energy Trade Liberalization**



I then report the correlation between the principal-component (PC) scores and the original data. As shown in Table 3C, the variable of number of FTA agreements signed or completed load heavily on C2 and C3. Similarly, Ratio of Energy Net Imports over Consumption draws heavily on C3 and C1. Energy Production significantly relies on C1 and C4. Total Energy Consumption is on C1 and C4. Per-capita Energy Consumption loads dramatically on C5 and C2. Finally, GDP per unit of energy use has significant weights on C4 and C3.

**Table 4C. Correlation between Raw Data and Calculated Eigenvectors for Energy Trade Liberalization**

Variable	C1	C2	C3	C4	C5	C6
Number of FTA Agreements Signed	0.252	0.534	0.318	-0.698	-0.251	-0.026
Ratio of Energy Net Imports over Consumption	0.194	-0.249	0.840	0.108	0.423	0.063
Energy Production (kt of oil equivalent)	0.575	0.215	-0.164	0.266	0.013	0.724
Total Energy Consumption (kt of oil equ.)	0.562	0.241	-0.041	0.407	-0.038	-0.676
Per-capita Energy Consumption (kg of oil equ.)	-0.302	0.633	-0.104	0.076	0.700	-0.013
GDP per unit of energy use	-0.401	0.384	0.392	0.509	-0.515	0.115

Source: Author's own calculation.

With these six eigenvalues at hand, I now pick the largest weight (i.e., 2.552) and use it to calculate the measured score for energy trade liberalization by country. Table 2D reports the score by country. Clearly, China, India, and Indonesia have higher extent of energy trade liberalization. This is due in large part to the large economic size of these countries. Economic size is captured implicitly by the indexes of total energy consumption and total energy production. This observation can be double-confirmed by observing that small countries like Brunei and Singapore have low scores of energy trade liberalization.

**Table 4D. Score of Energy Trade Liberalization, by using PCA Approach**

Country	Score	Country	Score	Country	Score
China	5.1034	Lao PDR	-0.1645	Japan	-0.4068
India	1.3743	Australia	-0.2799	Myanmar	-0.4884
Indonesia	0.6093	Malaysia	-0.3075	Philippines	-0.6024
Vietnam	0.1934	South Korea	-0.3319	Singapore	-1.8452
Thailand	-0.0770	New Zealand	-0.3474	Brunei	-2.3081
Cambodia	-0.1213				

Source: Author's own calculation.

## 4.2. Energy Infrastructure Development

As suggested in Shi and Kimura (2010), the extent of energy infrastructure is another important component of energy market integration in East Asia. To measure the energy infrastructure development in the region, the following indicators are chosen: (1) electric power consumption (kWh per capita); (2) road sector energy consumption (% of total energy consumption); and (3) road sector gasoline fuel consumption per capita (kt of oil equivalent). Electric power consumption is positively associated with energy infrastructure, though not a direct measure. In contrast, road sector energy consumption and road sector gasoline fuel consumption per capita are more tightly linked with the extent of energy infrastructure development, from both aggregate size and per-capita perspective. A potential indicator to measure energy infrastructure development is the length of gas pipelines in the ASEAN countries. Currently I leave this out from the calculation, but include it later as a robustness check. Table 5A summarizes the basic statistical information for the indicator mentioned above.

**Table 5. Summary Statistics for Indicators of Energy Infrastructure Development**

Variable	Mean	Std. Dev.	Min	Max
Electric power consumption (kWh per capita)	361.8	154.5	24	933
Road sector energy consumption	13.37	5.88	4	27
Road sector gasoline fuel consumption per capita	0.2	0.40	0	1

*Source:* Author's own calculation.

Similarly, I now calculate the predicted score for the eigenvalues for the three variables above by using the principal component analysis. In particular, I first calculate their covariance matrix of the three variables, and then obtain their eigenvectors and the associated eigenvalues. By sorting the eigenvalues from top to the bottom, table 5B demonstrates the eigenvalues of three eigenvectors associated with data on energy infrastructure development.

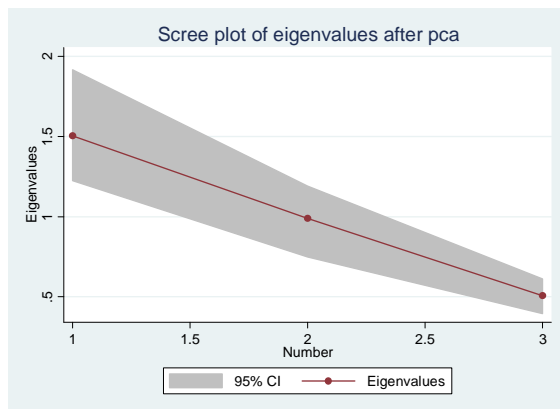
**Table 5B. The Eigenvalue for the PCA for Energy Infrastructure Development**

Component	Eigenvalue	Difference	Proportion	Cumulative
C1	1.5047	0.5154	0.5016	0.5016
C2	0.9893	0.4834	0.3298	0.8314
C3	0.5059	.	0.1686	1

Source: Author’s own calculation.

From Figure 3, one can observe that the highest eigenvalue (1.504) is around 3 times larger than the lowest eigenvalue (.505). Once again, I compute and report the heteroskedastic bootstrap confidence intervals for each eigenvalue.

**Figure 3. Plots of the Eigenvalues for Energy Infrastructure Development**



Turning to the correlation between the principal-component (PC) scores and the original data, Table 5C shows that the variable of electric power consumption loads heavily on C2. By contrast, the variable of road sector energy consumption loads heavily on C1 and C3. Finally, road sector gasoline fuel consumption per capita significantly relies on both C1 and C3.

**Table 5C. Correlation between Data and Calculated Eigenvectors for Energy Infrastructure Development**

Variable	C1	C2	C3	Unexplained
Electric power consumption (kWh per capita)	0.264	0.937	0.228	0
Road sector energy consumption	0.703	-0.025	-0.711	0
Road sector gasoline fuel consumption per capita	0.660	-0.348	0.667	0

Source: Author’s own calculation.

The last step is to calculate the score of the energy infrastructure development for each country, by adopting the eigenvector associated with the highest eigenvalue to multiply with the transposed standardized data. Table 5D reports the score by country. It seems that well-endowed resource countries such as New Zealand, Australia, and Brunei have a higher extent of energy infrastructure development. However, there is no predicted score for Lao PDR. In this sense, the comparison for this index is incomplete. I will address such a problem by using Wilberg's method shortly.

**Table 5D. Score of Energy Infrastructure Development, by using PCA Approach**

Country	Score	Country	Score	Country	Score
New Zealand	2.9688	Thailand	0.1086	India	-0.9024
Australia	1.9972	Indonesia	-0.1451	China	-1.3488
Brunei	1.3899	Japan	-0.2557	Myanmar	-1.4274
Philippines	0.8442	South Korea	-0.3772	Cambodia	-1.4274
Vietnam	0.2506	Singapore	-0.8630	Lao PDR	.
Malaysia	0.2301				

*Source:* Author's own calculation.

### 4.3. National Energy Market Liberalization

Shi-Kimura (2010) mention the positive and negative aspects of the policy landscape in each country. We now quantify each factor mentioned in their study. In particular, if the qualitative measure in Shi-Kumara (2010) is positive, I will assign a positive 1 point. Instead, if the qualitative measure is negative, a number of -1 is assigned to the factor. In addition, I adopt some other appropriate indicators which abstract from Shi-Kimura (2010). For example, I include index of nuclear energy (% of total energy use) and combustible renewable and wastes (% of total energy) inside. Based on this quantitative measure, I obtain the following data for the energy market integration in East Asia area as shown in Table 6A.

**Table 6A. Data on Energy Market Integration Generated by Qualitative Index of Shi-Kimura (2010)**

Country	Oil	Coal	Gas	Electricity	Nuclear Energy	Renewables
Australia	2	1	-1	4	1.3	4.3
Brunei	0	—	-3	-5	0.0	0.0
Cambodia	1	—	0	0	0.1	70.5
China	-3	1	-2	-1	3.2	9.9
India	1	-3	1	-2	2.7	27.2
Indonesia	-1	3	3	-3	3.7	27.5
Japan	3	3	2	1	15.3	1.4
Lao PDR	—	1	—	-2	—	—
Malaysia	-2	1	-2	-4	0.8	4.0
Myanmar	1	1	1	0	1.9	66.3
New Zeland	2	0	2	3	25.9	6.6
Philippines	2	—	-1	4	23.8	19.2
Singapore	3	—	2	2	0.0	0.0
South Korea	2	2	1	-3	16.9	1.2
Thailand	2	-1	3	-1	0.7	17.8
Vietnam	1	0	-2	3	4.6	44.0

*Source:* Author's own collection.

I then obtain the summary statistics in the following Table 6B. For the first five indexes, the maximum number for an economy is 4, which implies that the country has four positive aspects of achievements in energy market liberalization. In contrast, some countries have a score of -3, which implies that the country has three negative aspects in energy market liberalization.

**Table 6B. Summary Statistics for Indicators of Energy Market Liberalization**

Variable	Mean	Std. Dev.	Min	Max
Oil	0.9351	1.6918	-3	3
Coal	0.7324	1.4477	-3	3
Gas	0.2658	1.9137	-3	3
Electricity	-0.25	2.8636	-5	4
Nuclear energy	6.7624	8.7077	0	25.9
Renewables	19.690	22.641	0	70.5

*Source:* Author's own calculation.

Similar to before, I then calculate the eigenvectors and their associated eigenvalues of the six components by calculating their covariance matrix of the six variables, as shown in Table 6C.

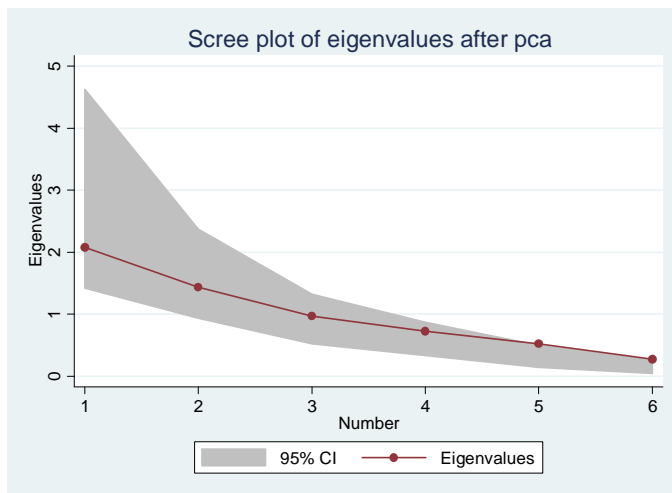
**Table 6C. The Eigen value for the PCA for Energy Market Liberalization**

Component	Eigen value	Difference	Proportion	Cumulative
C1	2.0785	0.6480	0.3464	0.3464
C2	1.4305	0.4624	0.2384	0.5848
C3	0.9682	0.2418	0.1614	0.7462
C4	0.7264	0.2030	0.1211	0.8673
C5	0.5233	0.2503	0.0872	0.9545
C6	0.2731	.	0.0455	1

Source: Author’s own calculation.

We then plot the six Eigen values from top to down in Figure 4. A 95% interval-confidence shaped area with heteroskedastic bootstrap is also drawn there. Here I also compute confidence intervals. Clearly, the highest Eigen value is 2.55 whereas the lowest one is .02.

**Figure 4. Plots of the Eigen values for the Energy Market Liberalization**



The next step is to calculate the correlation between the principal-component (PC) scores and the original data, Table 6D shows that the variable of oil market

liberalization loads heavily on C1. Data on coal market liberalization load significant on C4 whereas that of gas market liberalization is on C3 and that of electricity on C6. By contrast, the variable of nuclear energy consumption loans heavily on C5 whereas that of renewable resource loans heavily on C2 and C4.

**Table 6D. Correlation between Data and Calculated Eigenvectors for Energy Market Liberalization**

Variable	C1	C2	C3	C4	C5	C6	Unexplained
Oil	0.5953	0.155	0.1093	-0.1661	-0.3921	0.6545	0
Coal	0.0585	-0.6304	0.1642	0.7055	-0.2561	0.0944	0
Gas	0.3868	0.0754	0.7995	-0.0416	0.1429	-0.4281	0
Electricity	0.4921	0.2498	-0.4854	0.2561	-0.3096	-0.5462	0
Nuclear energy	0.4898	-0.2865	-0.274	0.0269	0.7646	0.1329	0
Renewable	-0.1027	0.6545	0.1055	0.6377	0.2826	0.2519	0

*Source:* Author's own calculation.

Finally, I calculate the score of the energy market liberalization for each country, by adopting the eigenvector associated with the highest Eigen value to multiply with the transposed standardized data. As shown in Table 4E, New Zealand has the highest level of energy market liberalization whereas Malaysia has the lowest level of energy market liberalization.

**Table 6E. Score of Energy Market Liberalization, by using PCA Approach**

Country	Liberalized Score	Country	Liberalized Score
New Zealand	2.3899	Lao PDR	-0.2326
Japan	1.9467	Myanmar	-0.2598
Philippines	1.8116	India	-0.5429
Singapore	1.1869	Cambodia	-0.6321
South Korea	0.7558	Indonesia	-0.7168
Australia	0.6226	Brunei	-2.0821
Thailand	0.3960	China	-2.1167
Vietnam	-0.1381	Malaysia	-2.3887

*Source:* Author's own calculation.

#### 4.4. Energy Pricing & Fossil Fuel Subsidies

Similar to before, to measure energy pricing, I follow the qualitative indicators mentioned in Shi and Kimura (2010). In particular, I assign a score of one point to the affirmative perspective on energy pricing in the market of oil, coal, and gas, respectively. In contrast, a negative one point is assigned if a country has bad performance. Moreover, I also consider the following three components in the analysis: (1) pump price for diesel fuel (US\$ per liter); (2) pump price for gasoline (US\$ per liter); (3) fossil fuel energy consumption (% of total). Based on these criteria, I generate the following data in Table 7A to measure the behaviour of energy pricing and subsidies.

**Table 7A. Data on Energy Pricing and Subsidies**

Country	Oil	Coal	Gas	Electricity	Diesel price	Gas price	Fossil fuel consumption (%)
Australia	1	1	1	0	0.94	0.74	94.4
Brunei	-1	-	-1	-2	0.21	0.38	100.0
Cambodia	-1	-	-1		0.89	0.94	29.1
China	-2	0	-1	0	1.01	0.99	86.9
India	-1	0	0	-2	0.70	1.09	70.0
Indonesia	-1	1	-1	-2	0.46	0.60	68.8
Japan	1	1	1	0	1.54	1.74	83.2
Lao PDR	-1		-1	-1	0.76	0.92	
Malaysia	-1		-1	-1	0.53	0.53	95.5
Myanmar	-1		-1	-1	0.52	0.43	31.7
New Zealand	1	1	1	1	0.85	1.09	67.4
Philippines	2	1	1	1	0.81	0.91	57.0
Singapore	1	-	1	1	0.90	1.07	100.0
South Korea	2	1	1	1			81.9
Thailand	-1	1	-1	-1	0.64	0.87	81.2
Vietnam	0	1	-1	-1	0.77	0.80	51.4

*Source:* Author's own calculation

Table 7B reports the main summary statistics for indicators of energy pricing liberalization.



**Table 7B. Summary Statistics for Indicators of Energy Pricing Liberalization**

Variable	Mean	Std. Dev.	Min	Max
Oil	-0.125	1.2583	-2	2
Coal	0.8062	0.3409	0	1.0279
Gas	-0.1875	0.9811	-1	1
Electricity	-0.4841	1.0895	-2	1
Diesel price	0.7689	0.2910	0.21	1.54
Gas price	0.8739	0.3226	0.38	1.74
Fossil fuel consumption (%)	73.006	22.047	29.1	100

*Source:* Author's own calculation.

Once again, I follow the “cook book” to examine the eigenvectors and their associated eigenvalues of the six components by calculating their covariance matrix of the six variables, as shown in Table 7C.

**Table 7C. The Eigen value for the PCA for Energy Pricing Liberalization**

Component	Eigen value	Difference	Proportion	Cumulative
C1	3.5639	2.0922	0.5091	0.5091
C2	1.4717	0.3357	0.2102	0.7194
C3	1.1360	0.6957	0.1623	0.8817
C4	0.4403	0.1644	0.0629	0.9446
C5	0.2760	0.2027	0.0394	0.9840
C6	0.0733	0.0346	0.0105	0.9945
C7	0.0387	.	0.0055	1

*Source:* Author's own calculation.

Similarly, Figure 5 then plots the Eigen value for each eigenvector following an declining trend of those Eigenvalues. The highest Eigen value reaches 6.1 whereas the lowest one only has a number of 0.2.

**Figure 5. Plots of the Eigenvalues for Energy Pricing Liberalization**

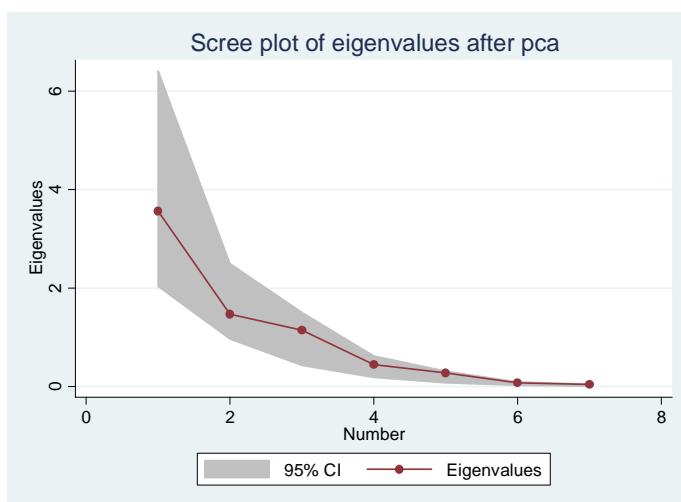


Table 7D reports the correlation between observed count data and the calculated eigenvectors for energy pricing liberalization. The most important component for oil pricing liberalization is C1 whereas the one for coal pricing liberalization is C2. Similarly, the most important component for gas pricing liberalization is C1 and that for electricity pricing liberalization is C5. Turning to diesel pricing continuum data, the most important component is C1. The one for gas pricing is C6. Finally, that for fuel consumption is C6 again.

**Table 7D: Correlation between Data and Calculated Eigenvectors for Energy Pricing Liberalization**

Variable	C1	C2	C3	C4	C5	C6	Unexplained
Oil	0.456	0.297	0.257	-0.074	-0.336	-0.002	-0.7206
Coal	0.138	0.712	-0.066	0.625	0.161	0.073	0.2182
Gas	0.485	0.015	0.245	-0.173	-0.463	-0.239	0.6349
Electricity	0.455	0.099	0.127	-0.442	0.704	0.260	0.0908
Diesel price	0.413	-0.261	-0.442	0.222	0.251	-0.659	-0.1424
Gas price	0.390	-0.352	-0.426	0.241	-0.222	0.660	0.0266
Fossil fuel consumption (%)	0.091	-0.451	0.690	0.520	0.199	0.030	-0.0277

*Source:* Author's own calculation.

Based on this, I then calculate the score of energy pricing liberalization by using the PCA approach (Table 7E). I find that Japan has the highest score on energy pricing liberalization, following South Korea, the Philippines, and New Zealand. In contrast, countries like Brunei and Indonesia have the lowest scores on energy pricing liberalization.

**Table 7E. Score of Energy Pricing Liberalization, by using PCA Approach**

Country	Liberalized Score	Country	Liberalized Score
Japan	3.4566	Vietnam	-0.6707
South Korea	2.1080	Lao PDR	-0.8987
Singapore	2.0943	Thailand	-1.0101
Philippines	2.0921	India	-1.0326
New Zealand	2.0466	Malaysia	-1.6417
Australia	1.4448	Myanmar	-1.9095
China	-0.6655	Indonesia	-2.0603
Cambodia	-0.6683	Brunei	-2.6850

*Source:* Author's own calculation.

#### 4.5. The Second-Step PCA Results

Thus far, I have calculated the predicted scores for the four categories to measure the energy market integration: (1) energy trade liberalization; (2) energy infrastructure development; (3) energy market liberalization; and (4) energy pricing liberalization. Table 8A describes the summary statistics for such variables:

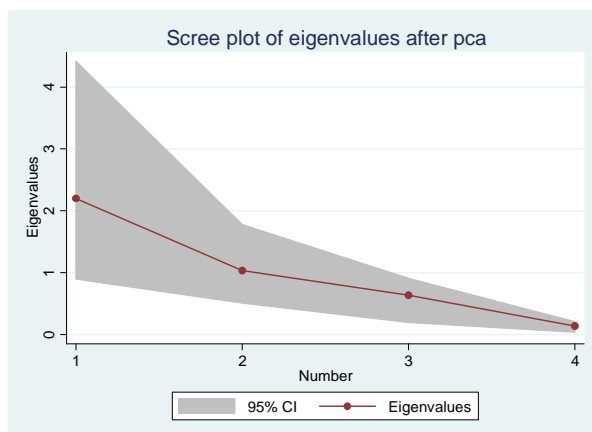
**Table 8A: Summary Statistics for the Four Variables**

Variable	Mean	Std. Dev.	Min	Max
Country Code	8.5	4.760	1	16
Energy Trade Liberalization	1.82E-08	1.597	-2.3081	5.1034
Energy Infrastructure Development	0.0695	1.286	-1.4274	2.9688
Energy Market Liberalization	1.02E-07	1.441	-2.3886	2.3899
Energy Pricing Liberalization	4.10E-08	1.8878	-2.6845	3.4566

*Source:* Author's own calculation

I now plot the four Eigenvalues in descending order, along with the heteroskedasticity robust confidence interval in Figure 6.

**Figure 6. Plots of the Eigenvalues for EMI Index**



The correlation of the four eigenvectors and the four predicted variables are summarized as follows:

**Table 8B. Correlation between Data and Calculated Eigenvectors for EMI Index**

Variable	C1	C2	C3	C4	Unexplained
Energy Trade Liberalization	-0.369	0.644	0.653	0.151	0
Energy Infrastructure Development	0.408	-0.519	0.750	-0.037	0
Energy Market Liberalization	0.617	0.286	-0.101	0.726	0
Energy Pricing Liberalization	0.563	0.484	-0.004	-0.670	0

*Source:* Author's own calculation.

Based on these information, I am now able to calculate the index of the energy market integration by country in Table 8C.

**Table 8C. Score of Energy Market Integration, by using PCA Approach**

Rank	Country	EMI Score	Rank	Country	EMI Score
1	New Zealand	2.5580	9	Brunei	-0.7296
2	Japan	1.7761	10	Cambodia	-0.9241
3	Philippines	1.7159	11	Myanmar	-1.0486

**Table 8C. (Continued)**

Rank	Country	EMI Score	Rank	Country	EMI Score
4	Australia	1.3289	12	Indonesia	-1.1205
5	Singapore	1.1947	13	India	-1.1606
6	South Korea	0.8364	14	Malaysia	-1.3673
7	Thailand	-0.1213	15	China	-2.6790
8	Vietnam	-0.2592			

*Source:* Author's own calculation

Clearly, New Zealand has the highest score of energy market integration (2.55), following by Japan, Philippines, Australia, Singapore, and South Korea. By contrast, China has the lowest score of energy market integration (-2.67), followed by Malaysia, India and Indonesia. The rest of the countries, which basically are the CLMV group, is located in between.

Although I have ranked almost all countries in East Asia area, the analysis omits Laos PDR. To include Laos PDR, I omit the index of energy infrastructure from the calculation, and re-perform the PCA analysis. Table 8D reports the modified ranking. Once again, Japan and New Zealand are the two countries with highest scores on EMI whereas China and Malaysia are the two countries with lowest scores. The CLMV group once again is caught in between.

**Table 8D: Score of Energy Market Integration, by using PCA Approach with Missing Data**

Rank	Country	EMI Score	Rank	Country	EMI Score
1	Japan	2.1941	9	Lao PDR	-0.3838
2	New Zealand	1.9079	10	Cambodia	-0.5031
3	Philippines	1.7034	11	Myanmar	-0.6749
4	Singapore	1.6706	12	India	-0.9018
5	South Korea	1.1514	13	Indonesia	-1.1751
6	Australia	0.8497	14	Brunei	-1.4201
7	Thailand	-0.1426	15	Malaysia	-1.6301
8	Vietnam	-0.3364	16	China	-2.3094

*Source:* Author's own calculation.

## 5. Concluding Remarks

Although East Asia is a relatively energy-abundant area in terms of its reserve and production, it still faces a challenge of insufficient energy supply problem due to the uneven energy allocation and high excess demand for energy. Therefore, intra-regional energy trade and further integration of energy markets is in urgent need. This in turn calls for a rigorous way to measure the current context of energy market integration in each country.

This paper provides such information by ranking of the extent of energy market integration for 16 East Asian countries, including the ASEAN 10 countries, China, Japan, Korea, India, Australia, and New Zealand. The extent of energy market integration (EMI) is measured by using a reliable statistical method -- the principal component analysis (PCA) approach. In particular, the score measuring the extent of EMI in each country is rooted by four important components: (1) energy trade liberalization; (2) energy infrastructure development; (3) energy market liberalization; and (4) energy pricing liberalization. Since each component also includes many sub-indicators, the final score of EMI in each country is conducted in two steps. I first calculate the measured score for each component. I then apply the PCA approach again to calculate the final scores of the extent of EMI.

My estimations show that countries like Japan and New Zealand have the highest extent of energy market integration. In contrast, countries like China and Malaysia, and India have lowest scores of EMI. The relatively poor ASEAN countries (i.e., the CLMV countries) are located in between. Such results are robust to different measures or different data adopted [I didn't find any different data in this analysis].

The policy implication for this finding is straightforward. Given that a further integrated energy market is good for each country, countries in East Asia area should try their every effort to foster their energy market integration. With the estimated score of EMI at hand, countries that have already lag behind the progress of energy market integration should work harder to catch up.

Several extensions and possible generalizations merit special consideration. One of them is to adopt the dynamic PCA method on panel data, to construct various indexes

as such data becomes available. Another possible extension is to have more indexes to enrich the measure of energy pricing and fossil fuel subsidies. These are some possible research topics to pursue in the future.

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## Appendix:

The Principal Component Analysis method is a popular approach to such analysis. Jolliffe (1986) is one of the first studies to systematically employ the PCA method. Rather than assigning an ad-hoc weight on each factor, the PCA method is able to find an appropriate weight for each component. In particular, the principal components sequentially capture the maximum variability among original data. It can guarantee minimal information loss, and hence is a good application to the real-world economic analysis. For example, Song and Sheng (2007) provide an interesting application to explain the economic growth after economic reform in 1979 in China.

The PCA method seeks the linear combinations of the original variables such that the derived variables capture maximal variance. In particular, as highlighted by Shlens (2005), it can be completed via singular value decomposition (SVD) of the data matrix. Let data  $X$  be a  $n \times p$  matrix, by demeaning the data, I obtain the eigen-arrays  $U$  which are the principal components (PCs) of unit length. Similarly, I can obtain the eigen-genes  $V$  are the corresponding loadings of the principal components. The first  $q$  ( $q \leq p$ ) PCs are chosen to represent the data. However, it is possible that I have missing data on some variables. In this case, I also have revised PCA approach, in particular, I use Wiberg's method.

We now go further to formally introduce the PCA approach.<sup>4</sup> In particular, consider a  $m \times n$  matrix  $Y = [y_1, y_2, \dots, y_n]$  and the mean of each vector is  $\bar{Y} = [\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n]$ , I first perform the demean process by defining  $X = [y_1 - \bar{y}_1, y_2 - \bar{y}_2, \dots, y_n - \bar{y}_n]$ .

The covariance of this matrix  $X$  is as follows:  $C_x = \frac{XX^T}{n-1}$  which is a squared symmetric  $n \times n$  matrix. The next step is to make the eigen-decomposition for the covariance matrix  $C_x$ . In particular, I need to calculate  $m$  dimensional eigen-vector  $E = [e_1, e_2, \dots, e_m]$  and their associated eigen-values  $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_m]$ . Note that  $C_x E = [C_x e_1, C_x e_2, \dots, C_x e_m] = [\lambda_1 e_1, \lambda_2 e_2, \dots, \lambda_m e_m]$ , where the second equality follows the property of eigen-values and eigen-vectors. Now I can transform the

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<sup>4</sup> Readers who are not interested in technical details can directly jump to the end of the section.

matrix as follow:

$$C_X E = [\lambda_1 e_1, \lambda_2 e_2, \dots, \lambda_m e_m] = \begin{bmatrix} e_{11} & e_{21} & \dots & e_{m1} \\ e_{12} & e_{22} & \dots & e_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ e_{1m} & e_{2m} & \dots & e_{mm} \end{bmatrix} \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_m \end{bmatrix} = ED,$$

where D denotes the eigen-value matrix. Hence, I obtain:  $C_X = EDE^1$ .

The next task is to find some orthonormal matrix P where  $Q=PX$ . I can show that this orthonormal matrix P indeed is the eigenvector matrix E. To see this more formally, consider  $Q = [q_1, q_2, \dots, q_n]$ , I have:  $Q = E^T X$  with its covariance matrix:

$$C_Q = \frac{QQ^T}{n-1} = \frac{E^T XX^T E}{n-1} = E^T \left( \frac{XX^T}{n-1} \right) E = E^T C_X E = E^T E D E^1 E = D,$$

where the second last equality comes from the relationship  $C_X = EDE^1$ , as shown above and the last equality holds due to the fact that the inverse of an orthogonal matrix is its transpose. That is, the covariance of the matrix  $C_Q$  indeed is a diagonal matrix.

The last step is to pick the eigenvector  $e_k$  from the eigenvectors matrix E which is associated with the largest eigenvalue  $\lambda_k$ . The new vector  $q_k = e_k^T X$ , which has  $1 \times n$  dimension, is the so-called principal component of the original vector X. In this way, the original  $m \times n$  dimensional matrix is reduced to a  $1 \times n$  dimensional matrix.

Now I can summarize the cook-book steps to obtain the principal components in a reader-friendly fashion. I first demean the raw data in a matrix form, followed by calculating its covariance. I then find the eigenvectors and associated eigenvalues for such a covariance matrix. Finally, I rank all the eigenvalues and pick the largest one. The last step obtains the principal component of the matrix, which is just the matrix constructed by multiplying the eigenvector with the highest eigenvalue and the original demeaned matrix.